Multiple Representation of Numbers

Lesson Synopsis:
Students will use a graphic organizer to identify and record multiple representations of two-digit numbers.

TEKS:

2.1 Number, operation, and quantitative reasoning. The student understands how place value is used to represent whole numbers. The student is expected to use concrete models to represent, compare, and order whole-numbers (through 999), read the numbers, and record the comparisons using numbers and symbols ( > , < , = ).

2.1A Use concrete models of hundreds, tens, and ones to represent a given whole number (up to 999) in various ways.

2.1B Use place value to read, write, and describe the value of whole numbers to 999.

Process TEKS:

2.13 Underlying processes and mathematical tools. The student communicates about Grade 2 mathematics using informal language.

2.13A Explain and record observations using objects, words, pictures, numbers, and technology.

2.13B Relate informal language to mathematical language and symbols.

2.14 Underlying processes and mathematical tools. The student uses logical reasoning.

2.14A The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.

GETTING READY FOR INSTRUCTION

Performance Indicator(s):
• Read, write, and describe the concrete representation of given numbers, and use place value to explain the relationship among the different representations on the given recording sheet. (2.1A, 2.1B) 1E, 2E, 2I, 3D, 3H, 4E, 5B, 5G

Key Understandings and Guiding Questions:
• Numbers can be represented with word, numerals, and manipulatives.
  — How can we group manipulatives to make it easier to calculate the value of a given two-digit number being represented?
  — What is the difference between the written representation of a number and the concrete representation of its value?
• The value of a number depends on the position of the digits.
  — How many tens and how many ones are represented in your model?
  — What value does each digit represent?

Vocabulary of Instruction:
• 10-long
• unit
• standard form
• place value
• expanded notation
• symbolic representation
• value
• overhead markers (optional)

Materials:
• number line
• base-ten blocks
• color tiles
• linking cubes
• counters
• number cubes/dice
• storage bags
Resources:

- Spiraling Review

Advance Preparation:

1. Pre-count 46 different manipulatives, place them into storage bags, and assign a different manipulative to each group/center (i.e., 46 counters, 46 color tiles, etc.).
2. Handout: Place Value Mat (1 per student)
3. Transparency: Place Value Mat (1 per teacher)
4. Handout: Multiple Representations Chart (laminate for continuous use with overhead pens or 3 per student)
5. Transparency: Multiple Representations Chart (1 per teacher)
6. Transparency: Base-Ten Drawing Graphic (1 per teacher)
7. Handout: Multiple Representations Practice Sheets (1 per student)

Background Information:

In Grade 1, students created sets of tens and one using concrete objects to represent numbers to 99.

GETTING READY FOR INSTRUCTION SUPPLEMENTAL PLANNING DOCUMENT

Instructors are encouraged to supplement, and substitute resources, materials, and activities to differentiate instruction to address the needs of learners. The Exemplar Lessons are one approach to teaching and reaching the Performance Indicators and Specificity in the Instructional Focus Document for this unit. A Microsoft Word template for this planning document is located at www.cscope.us/sup_plan_temp.doc. If a supplement is created electronically, users are encouraged to upload the document to their Lesson Plans as a Lesson Plan Resource in your district Curriculum Developer site for future reference.

INSTRUCTIONAL PROCEDURES

Instructional Procedures

ENGAGE

1. Divide students into small groups of four.
2. Distribute the pre-counted bag of 46 manipulatives to each group. Say to students, “I want you to guess the value of the mystery number contained within the bag and tell your fellow group members your guess.”
3. Students will look at the bag and guess the value of the number represented in the bag. Say to students, “Everyone open your mystery number bags and calculate the value of your mystery bag using the most efficient way possible.”
4. Select one person from each group or center to represent the number 46.
   - Which group would like to show and explain how you figured out the mystery value using your manipulatives? Ex: with color tiles.

Notes for Teacher

NOTE: 1 Day = 50 minutes
Suggested time: Day 1

Spiraling Review

MATERIALS

- pre-counted bag of 46 manipulatives (1 bag per group of four students)

TEACHER NOTE

As you observe the students, it is important to note students that are not unitizing manipulatives by 10s. This concept was a focal point of grade 1. Tier One reinforcement should take place to construct an understanding of unitizing in order to develop place value.
Instructional Procedures

Answers may vary. Our group made sets of tens to make it easier to count. We had four groups of ten and six individual ones left over.

- Can another group show and explain a different way of finding the mystery number using their manipulatives? Answers may vary according to manipulatives used.
- How are the methods used to find the mystery number the same? Answers may vary but should include counting groups of tens and ones.
- How are they different? Answers may vary according to manipulatives used.

EXPLAIN 1

1. Distribute one handout: Place Value Mat to each of the groups.

2. Explain that the Place Value Mat is a tool to help us identify the number of tens and ones in the value representation of a number.
   - Think about the number 46. Decide with your group how to represent 46 using the Place Value Mat and your manipulatives.

3. Referring to their previous concrete representation of number 46, have students place the sets of ten in the tens place value and the individual units in the ones place value on the Place Value Mat.

   - How many tens and ones are concretely represented on the place value mat? (4 tens and 6 ones)
   - What is the value of four tens? (40)
   - What is the value of six ones? (6)

4. Rotate students to another manipulative. Create a new two-digit number (less than 46 since the manipulatives are limited to that number). Write the new number on the board. Ask:
   - How can you concretely represent the value of the number on the Place Value Mat with your assigned manipulative?
   - How can you use the Place Value Mat to determine the number of tens and ones? (By placing the stacks of tens in the tens place and placing the ones in the ones place.)
   - What are the digits in each place value? Answers may vary according to the number written.
   - What value does each digit represent? Answers may vary according to the number written.
   - Did anyone group the number differently from the previous time? Answers may vary: We were able to represent a group of ten with one 10-long block this time, where last time we had to connect ten cubes to represent a group of ten.

Notes for Teacher

MATERIALS

- Handout: Place Value Mat (1 per student)
- Transparency: Place Value Mat (1 per teacher)

TEACHER NOTE

Students may try to state that they have “40 tens.” Correct the student by rephrasing that he or she has 4 tens which are valued at 40.
Instructional Procedures

EXPLAIN 2

1. Distribute laminated handout or 3 handouts: Multiple Representations Chart and Place Value Mat to each student. Give each group a bag of linking cubes or base-ten blocks. Students will be working with both handouts at the same time moving back and forth between the two.

2. Model how to use the multiple representations chart by writing the number “42” in the middle of the page. Explain to the students that the number “42” is a symbolic representation. Do not complete any other sections at this time.

3. Ask students to model the number “42” using their assigned manipulative and the Place Value Mat. Remind students that some groups are constructing their model using linking cubes while other groups using base-ten blocks. Ask:
   - Is it easier to use linking cubes or base-ten blocks? Explain why.

4. Display transparency: Base-Ten Drawing Graphic. Review with students how to draw a representation for a unit, ten and hundred.

5. Ask students to refer back to their Multiple Representation chart. Draw a representation of a concrete model of “42” in the pictorial portion of the chart. Ask students to sketch a picture of the model they have created. Invite several students to share their pictorial representations with the class. (See INSTRUCTIONAL NOTE)

6. Direct students: Compare the place value charts on both handouts.
   - Are they the same? (Yes)
   - How many tens are there in the number 42? (4)
     Teacher models how to record 4 on the place value chart.
   - How many ones are there in the number 42? (2)
     Teacher models how to record 2 on the place value chart.
   Describe the number of tens and ones directly below the place value chart.

Notes for Teacher

Suggested Day 2

SPIRALING REVIEW

MATERIALS
- Handout: Multiple Representations Chart (1 laminated per student or 2 paper copies)
- Transparency: Multiple Representations Chart (1 per teacher)
- Handout: Place Value Mat (1 per student)
- Linking cubes
- Base-ten blocks
- Transparency: Base-Ten Drawing Graphic

INSTRUCTIONAL NOTE

RESEARCH
According to the Principles and Standards for School Mathematics, encouraging students to explore and model relationships using language and notation will allow them to observe different relationships and make conjectures (generalizations) from their experiences with numbers.
7. Model how to represent the value of 4 tens and 2 ones in expanded notation as 40 + 2 by referring to the pictorial representation. Have students skip count by tens “10, 20, 30, 40”, and then write 40 in the Expanded Notation section. Now count the ones “1, 2” and then write 2 in the Expanded Notation section. 40 + 2 is known as the expanded notation of the number 42.
   - **What value does the digit 4 represent in the tens place? (40)**
   - **What value does the digit 2 represent in the ones place? (2)**
   - **How do you represent the expanded notation of this number? (40 + 2)**

8. Model how the expanded notation yields the written form of the number by writing forty-two as you read or say the number.

9. Continue to represent new numbers as guided practice. Walk around and visit with each group. Create opportunities for students to share their results from their chart.

**ELABORATE**

1. Distribute handouts: **Place Value Mat**, **Multiple Representations Chart**, and base-ten blocks to each student.

2. Ask students to represent the number 73 with their blocks. Record the pictorial on their handout. Once everyone has completed the model, ask students to work in pairs to complete their handout: **Multiple Representations Chart**.

3. Invite a student to share a section of their chart by completing a section on the overhead. Continue to select volunteers until the chart is complete.

4. Ask students to write 5 tens and 7 ones below the place value portion on their chart.

5. Once everyone has completed, ask students to work in pairs to complete their handout: **Multiple Representations Chart**.

6. Invite a student to share a section of their chart by completing a section on the overhead. Continue to select volunteers until the overhead chart is complete.

7. Review the components of the chart by asking the following questions:
   - **What does each section of the chart have in common? (They all represent the same number.)**
   - **What is different about each section? Responses may include:** The
### Instructional Procedures

pictorial or sketch does not show any digits, but demonstrates a picture of groups of ten and units of ones; the expanded notation uses numbers to show the value of each of the digits; or words are used to tell the value of the numbers in the written section of the chart, etc.

### EVALUATE

1. Distribute handout: **Multiple Representations Practice Sheet** to each student.

2. Students are to complete the missing components of the handout: **Multiple Representations Sheet** individually.

### MATERIALS

- Handout: **Multiple Representations Practice Sheet** (1 per student)
- Handout: Blank **Multiple Representations Chart** — can be used to create additional practice sheets (see ELABORATE section)

### TEACHER NOTE

This assessment is intended to evaluate the following Performance Indicator:

- Read, write, and describe the concrete representation of given numbers and use place value to explain the relationship among the different representations on the given recording sheet. (2.1A, 2.1B)
## Place Value Mat

<table>
<thead>
<tr>
<th>TENS</th>
<th>ONES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Multiple Representations Chart

Directions: Complete the charts below.

<table>
<thead>
<tr>
<th>PICTORIAL</th>
<th>PLACE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TENS</td>
</tr>
<tr>
<td>EXPANDED NOTATION</td>
<td></td>
</tr>
<tr>
<td>WRITTEN</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PICTORIAL</th>
<th>PLACE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TENS</td>
</tr>
<tr>
<td>EXPANDED NOTATION</td>
<td></td>
</tr>
<tr>
<td>WRITTEN</td>
<td></td>
</tr>
<tr>
<td>Name</td>
<td>Block</td>
</tr>
<tr>
<td>------------</td>
<td>---------------</td>
</tr>
<tr>
<td>Unit</td>
<td>(small cube)</td>
</tr>
<tr>
<td>10-Long</td>
<td><img src="image" alt="10-Long" /></td>
</tr>
<tr>
<td>100-Flat</td>
<td><img src="image" alt="100-Flat" /></td>
</tr>
</tbody>
</table>
Multiple Representations Practice Sheet (pp.1 of 3) KEY

Directions: Complete the charts below.

<table>
<thead>
<tr>
<th>PICTORIAL</th>
<th>PLACE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Pictorial" /></td>
<td><strong>TENS</strong></td>
</tr>
<tr>
<td><img src="image2" alt="Pictorial" /></td>
<td><strong>3</strong></td>
</tr>
<tr>
<td><strong>3 tens and 9 ones</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PICTORIAL</th>
<th>PLACE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Pictorial" /></td>
<td><strong>TENS</strong></td>
</tr>
<tr>
<td><img src="image4" alt="Pictorial" /></td>
<td><strong>4</strong></td>
</tr>
<tr>
<td><strong>4 tens and 1 one</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EXPANDED NOTATION</th>
<th>WRITTEN</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>30 + 9</strong></td>
<td><strong>thirty-nine</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EXPANDED NOTATION</th>
<th>WRITTEN</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>40 + 1</strong></td>
<td><strong>forty-one</strong></td>
</tr>
</tbody>
</table>
Multiple Representations Practice Sheet (pp 2 of 3) **KEY**

Directions: Complete the charts below.

<table>
<thead>
<tr>
<th>PICTORIAL</th>
<th>PLACE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>TENS</strong></td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td><strong>56</strong></td>
</tr>
</tbody>
</table>

**EXPANDED NOTATION**

50 + 6  
**fifty-six**

<table>
<thead>
<tr>
<th>PICTORIAL</th>
<th>PLACE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>TENS</strong></td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td><strong>69</strong></td>
</tr>
</tbody>
</table>

**EXPANDED NOTATION**

60 + 9  
**sixty-nine**
Multiple Representations Practice Sheet (pp. 3 of 3) **KEY**

Directions: Complete the charts below.

<table>
<thead>
<tr>
<th>PICTORIAL</th>
<th>PLACE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Pictorial" /></td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Pictorial" /></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TENS</th>
<th>ONES</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

8 tens and 3 ones

<table>
<thead>
<tr>
<th>Number</th>
<th>EXPANDED NOTATION</th>
<th>WRITTEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>83</td>
<td>80 + 3</td>
<td>eighty-three</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PICTORIAL</th>
<th>PLACE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4.png" alt="Pictorial" /></td>
<td></td>
</tr>
<tr>
<td><img src="image6.png" alt="Pictorial" /></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TENS</th>
<th>ONES</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

6 tens and 4 ones

<table>
<thead>
<tr>
<th>Number</th>
<th>EXPANDED NOTATION</th>
<th>WRITTEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>60 + 4</td>
<td>sixty-four</td>
</tr>
</tbody>
</table>
Multiple Representations Practice Sheet (pp. 1 of 3)

Directions: Complete the charts below.

<table>
<thead>
<tr>
<th>PICTORIAL</th>
<th>PLACE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>TENS</td>
<td>ONES</td>
</tr>
</tbody>
</table>

39

Number

EXPANDED NOTATION

WRITTEN

4 tens and 1 one

<table>
<thead>
<tr>
<th>PICTORIAL</th>
<th>PLACE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>TENS</td>
<td>ONES</td>
</tr>
</tbody>
</table>

4 1

Number

EXPANDED NOTATION

WRITTEN
Multiple Representations Practice Sheet (pp. 2 of 3)

Directions: Complete the charts below.

```
60 + 9
```
Multiple Representations Practice Sheet (pp. 3 of 3)

Directions: Complete the charts below.

PICTORIAL

PLACE VALUE

TENS  ONES

Number

EXPANDED NOTATION

WRITTEN

eighty-three

PICTORIAL

PLACE VALUE

TENS  ONES

Number

64

EXPANDED NOTATION

WRITTEN
Bibliography